NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA

National Certificate of Educational Achievement
TAUMATA MĀTAURANGA Ā-MOTU KUA TAEA

MATHEMATICS with CALCULUS

Level 3

90638 Manipulate real and complex numbers, and solve equations

Credits: Five

Answer ALL questions in the spaces provided in this booklet.

Show ALL working for ALL questions.

Check that this booklet has pages 2–9 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

For Assessor's use only **Achievement Criteria**		
Achievement	**Achievement with Merit**	**Achievement with Excellence**
Manipulate real and complex numbers, and solve equations. ☐	Solve more complicated equations. ☐	Solve problem(s) involving real or complex numbers. ☐
Overall Level of Performance ☐		

You are advised to spend 40 minutes answering the questions in this booklet.

QUESTION ONE

$u = 2\text{cis}\frac{\pi}{3}$ $v = 3\text{cis}\frac{\pi}{2}$ $w = 3 + 2\text{i}$ $z = 2 - 2\text{i}$

$p = 3 + 2\sqrt{3}$ $q = 2 - 3\sqrt{3}$

(a) Find $2w + \bar{z}$ leaving your answer in the form $a + b\text{i}$.

(b) Find $\frac{v}{u}$ leaving your answer in the form $r\text{cis}\theta$.

(c) Write u as a complex number in rectangular form.

(d) Write pq as a number in the form $a + b\sqrt{3}$.

QUESTION TWO

Find all the solutions (including complex number solutions) of each of the following equations.

(a) $x^3 - 2x^2 - 5x + 6 = 0$

(b) $\log_{10}(3x - 1) = 1.4$ giving your answer to 3 significant figures.

(c) $x^2 - 4x + 5 = 0$

QUESTION THREE

Find all the solutions of $z^4 = 81\text{cis}\frac{2\pi}{3}$ leaving your answers in polar form.

QUESTION FOUR

Solve $6^{x+1} = 3^x$ giving your answer to 3 significant figures.

QUESTION FIVE

Solve $\sqrt{2x-1} = x - 2$.

QUESTION SIX

$z = x + \mathrm{i}y$ is any non-zero complex number. If $z + \frac{1}{z} = k$, with k *real*,

(a) prove that either $y = 0$ or $x^2 + y^2 = 1$.

(b) Show that if $y = 0$ then $|k| \geq 2$